$O(m_d - m_u)$ Effects in CP-even and CP-odd $K \to \pi\pi$ Decays

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Strong isospin-breaking (IB) effects in CP-even and CP-odd $K \to \pi\pi$ decays are computed to next-to-leading order (NLO) in the chiral expansion. The impact of these corrections on the magnitude of the $\Delta I = 1/2$ Rule and on the size of the IB correction, Ω_{IB}, to the gluonic penguin contribution to ϵ'/ϵ are discussed.

In the presence of IB, the standard isospin decomposition of the $K^+ \to \pi^+\pi^0$, $K^0 \to \pi^+\pi^-, \pi^0\pi^0$ decay amplitudes, A_{+0}, A_{+-} and A_{00}, becomes [1]

$$\begin{aligned} A_{00} &= \sqrt{1/3}A_0 e^{i\Phi_0} - [\sqrt{2/3}]A_2 e^{i\Phi_2}, \\ A_{+-} &= \sqrt{1/3}A_0 e^{i\Phi_0} + [1/\sqrt{6}]A_2 e^{i\Phi_2}, \\ A_{+0} &= [\sqrt{3}/2]A'_2 e^{i\Phi'_2}. \end{aligned} \tag{1}$$

In the absence of the $I = 2$ component of electromagnetism (EM), the Φ_I are the $\pi\pi$ phases. In general, $|A'_2| \neq |A_2|$ due to EM- and strong-IB-induced $\Delta I = 5/2$ contributions. A_0, A_2 can be chosen real in the absence of CP violation.

Since $|A_0| \sim 20|A_2|$, IB "leakage" of the large octet amplitude into the $\Delta I = 3/2$ amplitude can be numerically significant. EM leakage contributions have been computed to NLO in Ref. [1]; we compute the NLO strong octet IB contributions. These enter Standard Model predictions of ϵ'/ϵ where the strong cancellation between gluonic penguin (O_6) and electroweak penguin (O_8) contributions is sensitive to the degree of strong-IB-induced suppression of the O_6 contribution [2].

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Table 1
Strong octet and EM IB leakage contributions in units of 10^{-6} MeV. The IC and LO IB fits yield $A_2 = A_2' = -2.1 \times 10^{-5}$ MeV and -2.4×10^{-5} MeV, respectively.

Source	$\delta^{(s)} A_2$	$\delta^{(s)} A_2'$
(8)	$(-1.56 \pm 0.63) + (0.42 \pm 0.05)\mathrm{i}$	$(-1.56 \pm 0.63) + (0.42 \pm 0.05)\mathrm{i}$
(EM)	$(-1.27 \pm 0.40) - (1.28 \pm 0.02)\mathrm{i}$	$(0.70 \pm 0.73) - (0.07 \pm 0.04)\mathrm{i}$

At leading chiral order (LO), the computation of the octet leakage contribution is unambiguous; the magnitude of the LO weak 27-plet low-energy constant (LEC) is *decreased* by $\Omega_{IB} = 13\%$. The corresponding O_6 suppression in ϵ'/ϵ is $1 - \Omega_{IB}$. Recent analyses of ϵ'/ϵ employ $\Omega_{IB} = 0.25 \pm 0.08$, the difference from the LO value reflecting estimates of the effect of η' mixing. This effect is NLO in the chiral expansion, but does not exhaust NLO contributions. A full NLO calculation can be performed using Chiral Perturbation Theory (ChPT). The importance of such a *complete* NLO determination can be seen from the recent discussion of NLO $\pi - \eta$ mixing effects [3]: the η' contribution (associated with the strong LEC L_7^r) turns out to be almost completely cancelled by a contribution proportional to L_8^r [3]. To compute the NLO IB leakage contributions one evaluates the tree and one-loop graphs of Ref. [4]. NLO tree contributions are either proportional to the product of the LO weak octet LEC $c^\pm$ and a single NLO strong LEC or proportional to one of the NLO weak LEC's. All loop graphs involve one vertex from the LO octet effective weak Lagrangian, $c^\pm Tr\left[\lambda^\pm \partial_\mu U^\dagger \partial^\mu U\right]$, where the superscripts $\pm$ label the CP even and odd cases, respectively, $\lambda^+ = \lambda_6$, $\lambda^- = \lambda_7$, and $U = exp\,(i\lambda \cdot \pi)$, is the usual matrix variable. The (scale-dependent) ratio of the sum of the loop contributions to the LO octet contribution for a given amplitude is thus completely fixed; the main uncertainty lies in a lack of knowledge of the NLO weak LEC's, for which we are forced to use models (see Refs. [4,5] for further discussion).

The contributions to A_2 and A_2' associated with EM [1] and octet IB [4] leakage are given in Table 1. The errors reflect uncertainties in the estimates of the NLO LEC's. Denoting the ratio of LO 27-plet to octet weak LEC's obtained neglecting, or including, IB by r_{IC}, or r_{IB}, respectively, we find $R_{IB} \equiv r_{IB}/r_{IC} = 0.963 \pm 0.029 \pm 0.010 \pm 0.034$. The errors reflect uncertainties in the weak NLO LEC combinations, the input value of $B_0(m_d - m_u)$, and the EM contributions, respectively. The deviation from 1 is significantly smaller than at LO (where $R_{IB} = 0.870$). The $\Delta I = 5/2$ contribution (dominantly EM in character [4]), leads to $|A_2|/|A_2'| = 1.094 \pm 0.039 \neq 1$, and significantly exacerbates the phase discrepancy problem for the neutral K decays [4].

For the CP-odd case, $\Omega_{IB} = \omega\, Im\, \delta A_2 /\, Im\, A_0$ ($\omega = Re\, A_0 / Re\, A_2 \simeq 22.2$; δA_2 is the octet leakage contribution). At LO, $\Omega_{IB} = 0.13 \equiv [\Omega_{IB}]_{LO}$. At NLO $\Omega_{IB} = [\Omega_{IB}]_{LO}\left[1 + \frac{\mathrm{Im}\,\delta A_2^{(NLO;ND)}}{\mathrm{Im}\,\delta A_2^{(LO)}} - \frac{\mathrm{Im}\,A_0^{(NLO;ND)}}{\mathrm{Im}\,A_0^{(LO)}}\right] \equiv [\Omega_{IB}]_{LO}\,[1 + R_2 - R_0]$. The superscript $(NLO;ND)$ indicates the sum of non-dispersion NLO contributions (involving NLO weak and strong LEC's and the non-dispersive parts of loop graphs). Neither the NLO $I=0$ IC nor NLO $I=2$ IB leakage CP-odd LEC combinations are known. The NLO dispersive contributions create phases consistent with Watson's theorem. Although the positive $I=0$ phases correspond to attractive FSI, NLO weak LEC corrections may, nonetheless, make $Im\,A_0$ smaller at NLO than the LO (see comments on Ref. [6] in Ref. [7] for a related discussion). If, however, NLO effects *do* enhance $Im\,A_0$ (decreasing the level of O_6-O_8 cancellation and increasing ϵ'/ϵ) Ω_{IB} will be simultaneously suppressed, further increasing ϵ'/ϵ. The known NLO contributions (loops and strong LEC terms) give contributions $-0.24(-0.31)$ to R_2 and $-0.02(+0.42)$ to R_0, at scale $\mu=m_\eta(m_\rho)$. Using the weak deformation model to estimate the weak NLO LEC's, $1+R_2-R_0=0.27$ while, for the chiral quark model, it lies between 0.62 and 1.42. Averaging these results, and taking their spread as a *minimal* indication of theoretical error, we thus obtain, at NLO, $\Omega_{IB}=0.11\pm 0.08$, a much smaller value than conventionally employed, though with comparable errors. It is also significantly smaller than the partial NLO estimate of Ref. [3], 0.16 ± 0.03, based on the strong LEC contributions only. The central value above, combined with conventional central values for the B-factors, leads to a $\sim 50\%$ increase in the predicted value for ϵ'/ϵ. To be conservative, we would propose using this lower central value with an even larger error estimate, in all future calculations of Standard Model values for ϵ'/ϵ.

References

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